

# UK Intermediate Mathematical Challenge 

## THURSDAY 5TH FEBRUARY 2015

## Organised by the United Kingdom Mathematics Trust

and supported by

Institute
and Faculty
of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: $\mathbf{1}$ hour.

No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; calculators and measuring instruments are forbidden.
4. Candidates in England and Wales must be in School Year 11 or below. Candidates in Scotland must be in S4 or below.
Candidates in Northern Ireland must be in School Year 12 or below.
5. Use B or HB pencil only. Mark at most one of the options $A, B, C, D, E$ on the Answer Sheet for each question. Do not mark more than one option.
6. Do not expect to finish the whole paper in 1 hour. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15. Six marks are awarded for each correct answer to Questions 16-25.

## Each incorrect answer to Questions 16-20 loses 1 mark. Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a dumb machine. Do not write or doodle on the sheet except to mark your chosen options. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to think, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. The UK IMC is about solving interesting problems, not about lucky guessing.

## The UKMT is a registered charity

1. What is the value of $1-0.2+0.03-0.004$ ?
A 0.826
B 0.834
C 0.926
D 1.226
E 1.234
2. Last year, Australian Suzy Walsham won the annual women's race up the 1576 steps of the Empire State Building in New York for a record fifth time. Her winning time was 11 minutes 57 seconds. Approximately how many steps did she climb per minute?
A 13
B 20
C 80
D 100
E 130
3. What is a half of a third, plus a third of a quarter, plus a quarter of a fifth?
A $\frac{1}{1440}$
B $\frac{3}{38}$
C $\frac{1}{30}$
D $\frac{1}{3}$
E $\frac{3}{10}$
4. The diagram shows a regular pentagon inside a square.

What is the value of $x$ ?
A 48
B 51
C 54
D 60
E 72

5. Which of the following numbers is not a square?
A $1^{6}$
B $2^{5}$
C $3^{4}$
D $4^{3}$
E $5^{2}$
6. The equilateral triangle and regular hexagon shown have perimeters of the same length.
What is the ratio of the area of the triangle to the area of the hexagon?

A 5:6
B 4:5
C 3:4
D 2:3
E 1:1
7. A tetrahedron is a solid figure which has four faces, all of which are triangles.
What is the product of the number of edges and the number of vertices of the tetrahedron?
A 8
B 10
C 12
D 18
E 24

8. How many two-digit squares differ by 1 from a multiple of 10 ?
A 1
B 2
C 3
D 4
E 5
9. What is the value of $p+q+r+s+t+u+v+w+x+y$ in the diagram?
A 540
B 720
C 900
D 1080
E 1440

10. What is the remainder when $2^{2} \times 3^{3} \times 5^{5} \times 7^{7}$ is divided by 8 ?
A 2
B 3
C 4
D 5
E 7
11. Three different positive integers have a mean of 7. What is the largest positive integer that could be one of them?
A 15
B 16
C 17
D 18
E 19
12. An ant is on the square marked with a black dot. The ant moves across an edge from one square to an adjacent square four times and then stops.
How many of the possible finishing squares are black?
A 0
B 2
C 4
D 6
E 8

13. What is the area of the shaded region in the rectangle?

A $21 \mathrm{~cm}^{2}$
B
$22 \mathrm{~cm}^{2}$
C $23 \mathrm{~cm}^{2}$
D $24 \mathrm{~cm}^{2} \mathrm{E}$ more information needed
14. In a sequence, each term after the first two terms is the mean of all the terms which come before that term. The first term is 8 and the tenth term is 26 . What is the second term?
A 17
B 18
C 44
D 52
E 68
15. A flag is in the shape of a right-angled triangle, as shown, with the horizontal and vertical sides being of length 72 cm and 24 cm respectively. The flag is divided into 6 vertical stripes of equal width.
What, in $\mathrm{cm}^{2}$, is the difference between the areas of any two adjacent stripes?
A 96
B 72
C 48
D 32
E 24
16. You are asked to choose two positive integers, $m$ and $n$ with $m>n$, so that as many as possible of the expressions $m+n, m-n, m \times n$ and $m \div n$ have values that are prime. When you do this correctly, how many of these four expressions have values that are prime?
A 0
B 1
C 2
D 3
E 4
17. The football shown is made by sewing together 12 black pentagonal panels and 20 white hexagonal panels. There is a join wherever two panels meet along an edge.
How many joins are there?

A 20
B 32
C 60
D 90
E 180
18. The total weight of a box, 20 plates and 30 cups is 4.8 kg . The total weight of the box, 40 plates and 50 cups is 8.4 kg . What is the total weight of the box, 10 plates and 20 cups?
A 3 kg
B 3.2 kg
C 3.6 kg
D 4 kg
E 4.2 kg
19. The figure shows four smaller squares in the corners of a large square. The smaller squares have sides of length $1 \mathrm{~cm}, 2 \mathrm{~cm}, 3 \mathrm{~cm}$ and 4 cm (in anticlockwise order) and the sides of the large square have length 11 cm .
What is the area of the shaded quadrilateral?

A $35 \mathrm{~cm}^{2}$
B $36 \mathrm{~cm}^{2}$
C $37 \mathrm{~cm}^{2}$
D $38 \mathrm{~cm}^{2}$
E $39 \mathrm{~cm}^{2}$
20. A voucher code is made up of four characters. The first is a letter: V, X or P. The second and third are different digits. The fourth is the units digit of the sum of the second and third digits. How many different voucher codes like this are there?
A 180
B 243
C 270
D 300
E 2700
21. A rectangle is placed obliquely on top of an identical rectangle, as shown.
The area $X$ of the overlapping region (shaded more darkly) is one eighth of the total shaded area.
What fraction of the area of one rectangle is $X$ ?
A $\frac{1}{3}$
B $\frac{2}{7}$
C $\frac{1}{4}$
D $\frac{2}{9}$
E $\frac{1}{5}$

22. The diagram shows a shaded region inside a large square. The shaded region is divided into small squares.
What fraction of the area of the large square is shaded?
A $\frac{3}{10}$
B $\frac{1}{3}$
C $\frac{3}{8}$
D $\frac{2}{5}$
E $\frac{3}{7}$

23. There are 120 different ways of arranging the letters, U, K, M, I and C. All of these arrangements are listed in dictionary order, starting with CIKMU. Which position in the list does UKIMC occupy?
A 110 th
B 112 th
C 114 th
D 116 th
E 118 th
24. In square $R S T U$ a quarter-circle arc with centre $S$ is drawn from $T$ to $R$. A point $P$ on this arc is 1 unit from $T U$ and 8 units from $R U$. What is the length of the side of square RSTU?
A 9
B 10
C 11
D 12
E 13

25. A point is marked one quarter of the way along each side of a triangle, as shown.
What fraction of the area of the triangle is shaded?
A $\frac{7}{16}$
B $\frac{1}{2}$
C $\frac{9}{16}$
D $\frac{5}{8}$
E $\frac{11}{16}$


